Functional renormalization group for few-body physics

Hierarchy in FRG & superselection rule

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Introduction & Motivation

Functional renormalization group (FRG)

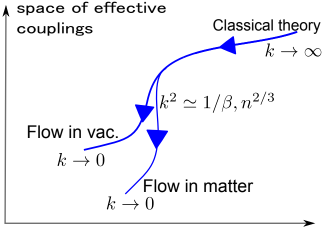
Basic objects:

- Γ_k: low-energy 1PI effective action with a scale $\sim k^2/2m$
- R_k: infrared regulator suppressing low-energy modes

Wetterich equation (Wetterich 1993):

$$\partial_k \Gamma_k[\Phi] = \frac{1}{2}\mathrm{STr}\left[\frac{\partial_k R_k}{\Gamma_k^{(2)}[\Phi] + R_k}\right]$$

($\Gamma_k^{(2)} = \delta^2\Gamma_k/\delta\Phi\delta\Phi$: field-dependent propagator)



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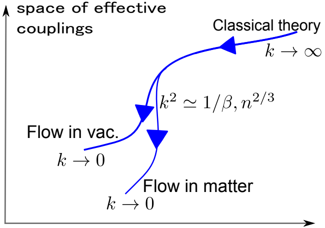
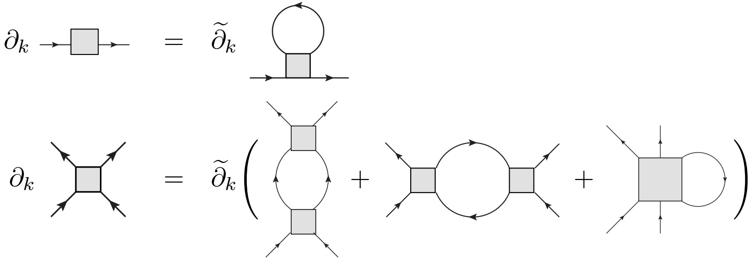


Features of FRG:

- Non-perturbative approach of quantum field theories
- Practical realization of Wilsonian RG
- Unified description of few-body and many-body physics

1-loop property of Wetterich equation

Wetterich equations for self-energy and four-point functions:



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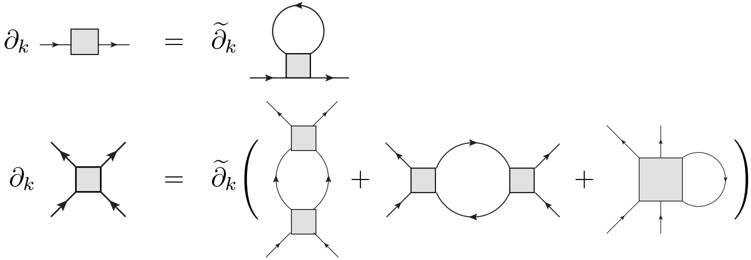

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Important!

- *1PI property + functional realization of RG $\Rightarrow$ exact 1-loop property*
- *Flow of a $2n$-point vertex depends on $2(n+1)$-point vertices (Hierarchy of FRG)*

Schrödinger equation

n-body Schrödinger equation ($|\psi\rangle$: n-body wave function):

$$i\hbar\frac{\partial}{\partial t}|\psi\rangle = (\hat{H}_0 + \hat{V})|\psi\rangle$$

Hamiltonian manifestly commutes with the number operator $\hat{N}$:

$$[\hat{H}_0, \hat{N}] = [\hat{V}, \hat{N}] = 0$$

Important!

Particle number operator $\hat{N}$ is a superselection charge.
$\Rightarrow$*n-body scattering amplitude does not depend on $(n+1)$-body physics*

Hierarchy of FRG in few-body physics

Hierarchy of FRG

Flow of a $2n$-point vertex function depends on $2(n+1)$-point vertices

However, we must be able to solve this hierarchy for few-body physics rigorously!
$\Rightarrow$ We can construct the closed flow equation.

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Purpose of this talk

Construction of the closed flow equation for few-body physics.

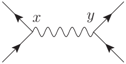
Solution of the hierarchy
&
Construction of the closed flow equation

Nonrelativistic model

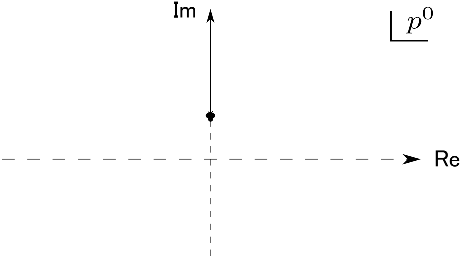
Propagators (m:mass, μ:chemical potential (<0)):

$$= \frac{1}{ip^0 + \boldsymbol{p}^2/2m - \mu + R_k(\boldsymbol{p})}$$

Interactions:

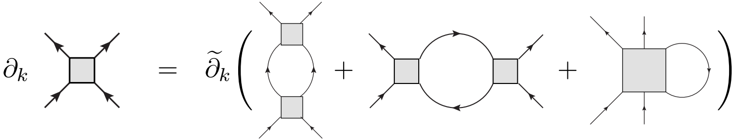


$$= \delta(x^0 - y^0) V(\boldsymbol{x} - \boldsymbol{y})$$

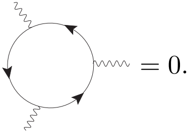


Two-body physics

Flow equation for four-point vertex functions:



Each diagram in second and third terms must contain sub-diagrams such as



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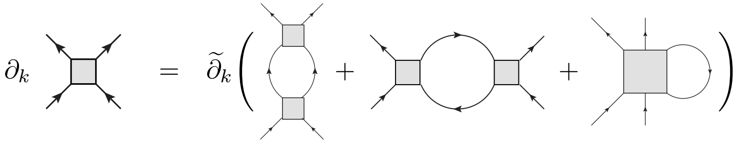

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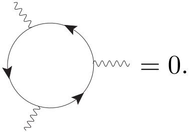
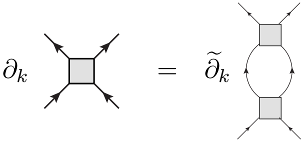
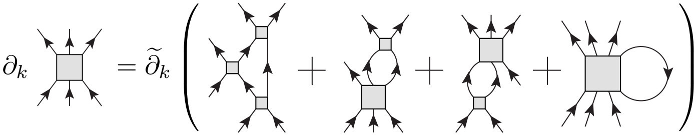


$\Rightarrow$ Second and third terms vanish, and hierarchy is solved for two-body physics:



Three-body physics

Flow of six-point vertex functions:



Does the last term vanish as in the case of two-body physics?

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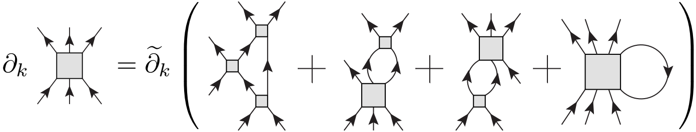


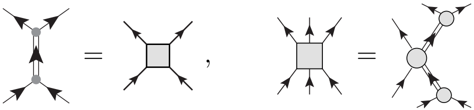
Does the last term vanish as in the case of two-body physics?

Ans.: No. If we put the last diagram to be zero, we would lose

- consistency with other non-perturbative approach, such as the Dyson-Schwinger (DS) equation,
- universality of renormalization group, such as arbitrariness of R_k, etc.

Substructure of eight-point vertex functions

Let us introduce another notation for four-point vertex
and following decomposition of six-point vertices:



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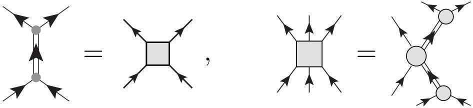
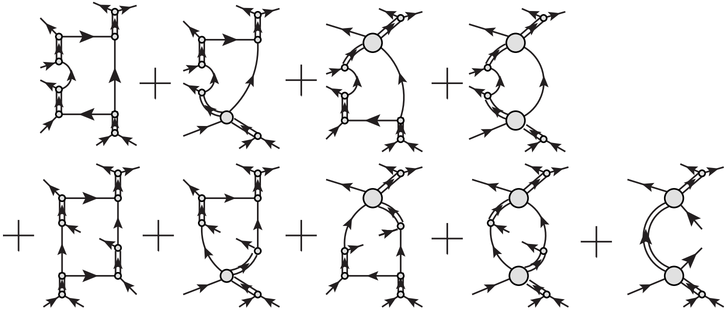
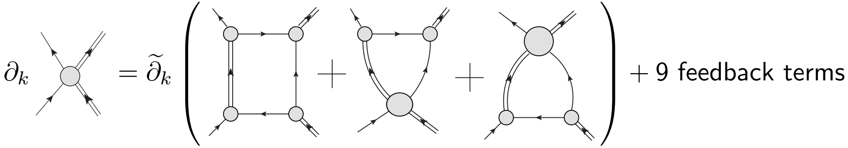


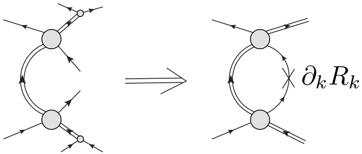
Subset of diagrams in the eight-point vertex functions:



Closed flow equation for three-body scattering problem



Feedbacks are obtained as follows:



Important!

The closed *and* exact *flow equation for three-body physics is constructed.*
$\Rightarrow$ *The solution of this closed flow equation can be shown to satisfy the DS eq.*

Summary & Perspectives

Summary

- FRG provides a unified description for few- and many-body physics.
- Hierarchy in FRG can be solved rigorously for few-body physics.
- Construction of the closed flow equation is completed for two- and three-body physics

Perspectives

- General construction of the closed flow equation.
- Application to Efimov physics of the exact and closed flow equation
- Application of knowledge of three-body physics to many-body properties